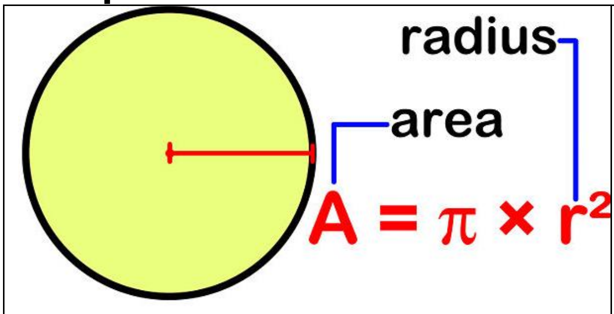
POLYNOMIALS (Unit 5) for 3th ESO BILINGUAL MATHS

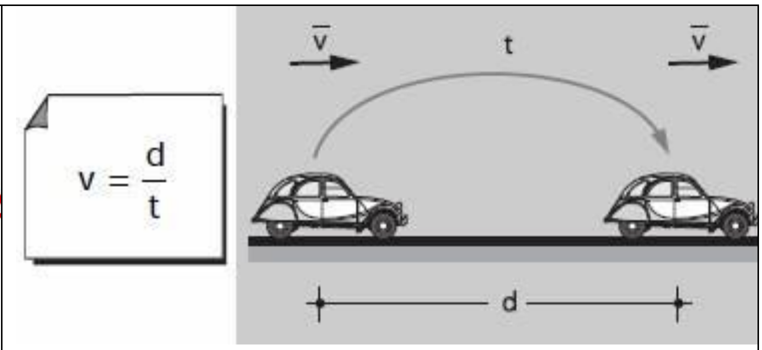
1. ALGEBRAIC EXPRESSIONS

An algebraic expression is any expression containing letters, numbers and the algebraic operations (addition, subtraction, multiplication ...) Letters are **constants, variables or unknowns**.

Examples:



Area of a circle



More examples:

The sum of the squares: $a^2 + b^2$

Double one number subtracted from triple another number: $3x - 2y$

The sum of different powers of a number: $a^4 + a^3 + a^2 + a$

Exercises:

1. Write like an algebraic expression:

   a. The area of a rectangular courtyard whose sides are x and y:
   b. The volume of a cube whose edge is a:
   c. A multiple of 7:
   d. The square of the hypotenuse (h) is equal to the sum of the squares of the other two sides (a and b)**:**
   e. The quarter part of a number plus twice the number:
   f. Denelle planted 22 fewer trees than Shelby. Shelby planted *t* trees.
   g. Three consecutive numbers:
   h. Three odd consecutive numbers:

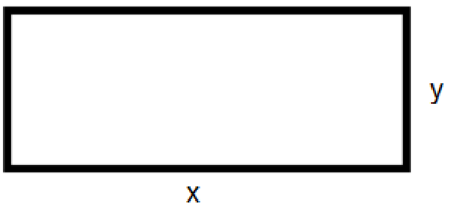
2. Describe the following algebraic expressions:

   a. $-x^5$
   b. $A = \pi r^2$
   c. $3\sqrt{y+1}$

d. $2x - \frac{x}{3}$

2. NUMERICAL VALUE OF AN ALGEBRAIC EXPRESSION:

The numerical value of an algebraic expression is the numerical value we obtain when we substitute letters for their numerical values and carry out the operations.



Example:

a)Write the perimeter formula:
b)Calculate the numerical value of the perimeter when x = 7 and y = 4:
c)Calculate the numerical value of the perimeter for x = $\frac{1}{2}$ and y = $\frac{1}{4}$:

3. EQUIVALENT EXPRESSIONS:

Two algebraic expressions are said to be equivalent when the same numerical value is obtained for any value of the variables.
To obtain an equivalent expression, you can to multiply and divide by the same number, or use any mathematical property.
Example:

$$\boldsymbol{x^2 + 3x + \frac{1}{4} \equiv \frac{4x^2 + 12x + 1}{4}}$$

Demonstrate that both expressions are equivalent substituting x for 4:

- Numerical value of $x^2 + 3x + \frac{1}{4}$ for x=4:
- Numerical value of $\frac{4x^2+12x+1}{4}$ for x =4:

Exercises:

4. Find the numerical value of the algebraic expression $\boldsymbol{a^2 - 3ab + 2}$ when:

   a) a=2 and b=3

   b) a=-1 and b=2

5. Obtain the numerical value of the following algebraic expressions to check whether they are equivalent:

a. $(x+y)^2 = x^2 + y^2$
b. $(x-y)^2 = x^2 + y^2 - 2xy$
c. $(x+y)\cdot(x-y) = x^2 - y^2$
d. $x^3 = 3x$
e. $x^2 + x^2 + x^2 = 3x^2$

6. Jane spent \$42 for shoes. This was \$14 less than twice what she spent for a blouse. How much was the blouse?

7. There are *b* boys in the class. This is three more than four times the number of girls. How many girls are in the class?

8. The sum of two consecutive numbers is 37. What are they?

9. One number is 10 more than another. The sum of twice the smaller plus three times the larger, is 55. What are the two numbers?

10. Divide \$80 among three people so that the second will have twice as much as the first, and the third will have \$5 less than the second.

11. A group of 266 persons consists of men, women, and children. There are four times as many men as children, and twice as many women as children. How many of each are there?

4. MONOMIALS

- A **monomial** is the product of a real number (**coefficient or numerical part**) and one or more letters or variables (**literal part**) using only the operation of multiplication and natural exponents.

- The **degree** of the monomial is the sum of the exponents on variables.

  Example: $-2x^2$; $\frac{1}{6}x$; $-5x^3y^4$; x^5y are four monomials whose degrees are 2, 1, 7 and 6 respectively

- Two monomials are **like** when they have the same literal part.

  Example: $2ax^4y^3$; $-3ax^4y^3$; ax^4y^3 ; $5ax^4y^3$ are like monomials whereas:

  axy^3 ; $3a^2x^4y^3$; $2bx^4$ are not like monomials.

- Only like monomials can be added together.

  Example: $5ax^4y^3 - 2ax^4y^3 = 3ax^4y^3$

  $4ax^4y^3 + a\,x^4y^3 =$

- To multiply monomials we multiply the coefficients of each term and also the literal parts.

  Example: $4ax^4y^3 \cdot (-x^2y) \cdot 3ab^2y^3 = -12a^2b^2x^6y^7$

- To divide monomials we divide the coefficients and also the literal parts:

  Example: $\frac{8x^5y}{2x^3} = 4x^2y$

Exercises:

12. Indicate whether the following expressions are monomial:

    a. $\frac{1}{5}x$
    b. $3x^{-5}$
    c. $-6x^{\frac{1}{3}}$
    d. $\frac{4}{x}$

13. Find the sum of these monomials:

    a) $2ax^4 - 3ax^4 + 5ax^4$

    b) $2x^3 - x + x^3 + 3x^3 + 2x$

14. Find the product of the following monomials: $2ax^2 \cdot (-3a^3x) \cdot 5y^4x^3$

15. Find $6a^5x^2y : 2a^3x$

16. Calculate $6a^5x^2y : 3a^6x$ (Is the answer a monomial this time? Why/why not?)

3. POLYNOMIALS

- A **polynomial** is the sum of two or more monomials.
  The monomials that form the plynomial are called **terms.**
  The **degree** of a polynomial is the degree of the term with the highest degree.

  Example:

Polynomial	Number of Degree	Classification
5	0	Constant
3x	1	Linear or First
$2x^2 + x - 4$	2	Quadratic or Second
$6x^3 - x + 24x^3 - 1$	3	Cubic or Third
$x^4 + 6x^3 - x + 2$	4	Quartic or Fourth

- To add or subtract two polynomials you add or subtract like terms.
- To multiply a monomial by a polynomial, you multiply the monomial by each of the polynomial's terms.
- To multiply two polynomials you multiply each term in one polynomial by each term in the other, and then add the like monomials.

Examples:

$$\left(4x^2+3x-14\right)-\left(x^3-x^2+7x+1\right)=$$
$$=4x^2+3x-14-x^3+x^2-7x-1$$
$$=-x^3+\left(4x^2+x^2\right)+\left(3x-7x\right)+\left(-14-1\right)$$
$$=-x^3+5x^2-4x-15$$

$$\left(4x^2+3x-14\right)\cdot\left(7x+1\right)=$$
$$4x^2\left(7x+1\right)+3x\left(7x+1\right)-14\left(7x+1\right)=$$
$$28x^3+4x^2+21x^2+3x-98x-14=$$
$$28x^3+25x^2-95x-14$$

Exercises:

17. We have the polynomials:

$A(x)=x^3-7x^2+5$; $B(x)=-3x^5+2x^3-x^2+5x$; $C(x)=\frac{1}{2}x^3-x$; $D(x)=-x^2$

Calculate:

a. A(x)+B(x)
b. B(x)-A(x)
c. C(x)-B(x)
d. A(x)+4C(x)
e. B(x)·D(x)

f. A(x)·C(x)
g. [A(x)-5]:D(x)

4. Special binomial products

$(a \pm b)^2 = a^2 \pm 2 \cdot a \cdot b + b^2$

$(x + 3)^2 = x^2 + 2 \cdot x \cdot 3 + 3^2 = x^2 + 6x + 9$; $(2x - 3)^2 = (2x)^2 - 2 \cdot 2x \cdot 3 + 3^2 = 4x^2 - 12x + 9$

$(a + b) \cdot (a - b) = a^2 - b^2$; $(2x + 5) \cdot (2x - 5) = (2x)^2 - 5^2 = 4x^2 - 25$

Exercises:

18. Expand the following expressions:
    a. $(2x + 6)^2$
    b. $(xy + 5x)^2$
    c. $(2x^4 - 8)^2$
    d. $(\frac{1}{4}x^4 - 8)^2$

19. Find the product of:
    a. $(2x^2 + 9)(2x^2 - 9)$
    b. $(3xy + y^2)(3xy - y^2)$

20. Write like a binomial product:
    a. $4x^4 + 20x^2 + 25$
    b. $9x^9 - 1$
    c. $x^4 - 25$
    d. $x^6 - 1$