NUMBER SEQUENCES

A **sequence** is a list of numbers in a certain order. Each number in a sequence is called a **term**.

- An ***arithmetic sequence*** is a sequence of numbers each of which, after the first, is obtained by adding to the preceding number a constant number called the ***common difference****.*

  Examples:
  3, 8, 13, 18, 23,... is an arithmetic sequence because each term is obtained by adding 5 to the preceding number. The common difference is 5.
  3 is the $\mathbf{a_1}$ term (the first term), 8 is the $\mathbf{a_2}$ term (second term) and so on.

  More examples:

- A ***geometric sequence*** is a sequence of numbers each of which, after the first, is obtained by multiplying the preceding number a constant number called the ***common rate****.*

  Example:
  3, 6, 12, 24, 48,... is a geometric sequence because each term is obtained by multiplying the preceding number by 2. The common rate is 2.

  More examples:

Arithmetic sequences

If the initial term of an arithmetic sequence is $\mathbf{a_1}$ and the common difference of successive members is ***d***, then:

$$a_2 = a_1 + d$$
$$a_3 = a_2 + d = a_1 + 2d$$
$$a_4 = a_3 + d = a_1 + 3d$$

$$\mathbf{a_n = a_1 + (n - 1)d \rightarrow \text{General expression}}$$

To solve exercises using arithmetic sequences you need the following formulas:

- The ***n*th** term: $\mathbf{a_n = a_1 + (n - 1)d}$ where:

  a_1 = the first term of the sequence
  d = common difference

You can obtain it using $\boldsymbol{d = a_n - a_{n-1}}$

or $$\boldsymbol{d = \frac{a_n - a_1}{(n-1)}}$$

n = number of terms
$\boldsymbol{a_n}$ = *n*th term (term occupying the nth place)

Example:

Given the arithmetic sequence 11, 7, 3,... Find the 301st term, a_{301}

Using the formula $a_n = a_1 + (n – 1)d$

$a_{301} = 11 + (301 – 1) \cdot (- 4) = - 1189$

Exercises:

1. Write the general expression to describe the sequence -3, -6, -9, -12,...

2. Write an expression to describe the sequence -7, -6, -5, -4,... Use n to represent the position of a term in the sequence, where n=1 for the first term.

3. Given the arithmetic sequence in which $\boldsymbol{a_2 = 7}$ and $\boldsymbol{a_6 = -5}$, obtain the three first term of the sequence. Hint: First, calculate **d**.

Geometric sequences

A ***geometric sequence*** is a sequence of numbers each of which, after the first, is obtained by multiplying the preceding number by a constant number called the ***common rate****.* Look at this:

$$a_1$$
$$a_2 = a_1 \cdot r$$
$$a_3 = a_2 \cdot r = a_1 \cdot r^2$$
$$a_4 = a_3 \cdot r = a_1 \cdot r^3$$
$$\dots$$

$$\mathbf{a_n = a_1 \cdot r^{n-1} \rightarrow \text{General expression}}$$

3, 6, 12, 24, 48,... is a geometric sequence because each term is obtained by multiplying the preceding number by 2.

To solve exercises using geometric sequences you need the following formula:

The *n*th term: $\mathbf{a_n = a_1 \cdot r^{n-1}}$

where:
a_1 = the first term of the sequence
r = common rate
n = number of terms
a_n = nth term

Example 1: Given 27, -9, 3, -1, ... Find an and a_8

Using $\mathbf{a_n = a_1 \cdot r^{n-1}}$

$$a_n = 27 \cdot \left(-\frac{1}{3}\right)^{n-1} \quad a_8 = 27 \cdot \left(-\frac{1}{3}\right)^{8-1} = 3^3 \cdot \frac{-1}{3^7} = -\frac{1}{3^4} = -\frac{1}{81}$$

Example 2: Given a geometric sequence with $a_2 = -10$ and $a_5 = -80$, find a_n.

$\mathbf{a_n = a_1 \cdot r^{n-1}}$ so we need to find a_1.

To find it we use the next system of equations:

$$\left.\begin{array}{r} -10 = a_1 \cdot r \\ -80 = a_1 \cdot r^4 \end{array}\right\}$$

solving by substitution:

$$a_1 = \frac{-10}{r} \qquad , \qquad -80 = \frac{-10}{r} \cdot r^4 \Rightarrow -80 = -10r^3 \Rightarrow r^3 = 8 \Rightarrow r = 2$$

and $a_1 = -5$.
That is $a_n = -5 \cdot 2^{n-1}$

Exercise: Given a geometric sequence where a_1= 5 and a_4= 320, obtain the general expression to calculate any term.

Evaluate variable expressions for number sequences

1. Find the first four terms of the sequence a_n = 4n - 3, where n represents the position of a term in the sequence. Start with n=1.

2. Find the first four terms of the sequence 2n, where n represents the position of a term in the sequence. Start with n=1.

Word problems

1) You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on in an arithmetic sequence. What is the total distance the object will fall in 6 seconds?
an = a1 + (n - 1) d

an = 16 + (6 – 1) 32 = 176

The 6th term is 176. Now we are ready to find the sum:

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_6 = \frac{6(16+176)}{2} = 576 \;\; feet$$

2) After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it before you are up to jogging 60 minutes per day.