

# *MOZART DICE GAME*



**MUSIC & MATHS**

**Johannes Chrysostomus  
Wolfgangus Theophilus  
Mozart**



# Johannes Chrysostomus Wolfgangus Theophilus Mozart



# Mozart





# Mozart

## (1756-1791)

- ▶ Born in Salzburg (Austria).
- ▶ Violin and piano master.
- ▶ He composed from the age of five. 😲
- ▶ Early death at the age of 35 😞



- ▶ He composed more than 600 works.
- ▶ He is considered among the greatest classical composers of all time.

# MUSIKALISCHES WÜRFELSPIEL

This is an interesting musical game done by **Mozart** when he was 21 years old.

The basis of the musical dice game consists of 176 musical measures given a certain dice roll. The result is a randomly selected 16 bar minuet and 16 bar trio.



► These  
are  
the  
176  
bars

TABLE de MUSIQUE. 3.

The image displays a musical score titled "TABLE de MUSIQUE." with the number "3." in the top right corner. The score is written in a 3/4 time signature and consists of 48 numbered bars, arranged in six systems of two staves each (treble and bass clef). The notation includes various rhythmic values, accidentals, and dynamic markings such as *l<sup>ro</sup>* and *2.* The bars are numbered sequentially from 1 to 48, with some bars containing multiple measures. The score concludes with a final double bar line.

► These  
are  
the  
176  
bars

Musical notation for measures 49-56. The score is written for a grand staff (treble and bass clefs). Measures 49-56 are indicated by numbers below the staff.

Musical notation for measures 57-64. The score is written for a grand staff (treble and bass clefs). Measures 57-64 are indicated by numbers below the staff.

Musical notation for measures 65-72. The score is written for a grand staff (treble and bass clefs). Measures 65-72 are indicated by numbers below the staff.

Musical notation for measures 73-80. The score is written for a grand staff (treble and bass clefs). Measures 73-80 are indicated by numbers below the staff.

Musical notation for measures 81-88. The score is written for a grand staff (treble and bass clefs). Measures 81-88 are indicated by numbers below the staff. A first ending bracket is shown under measures 81-88, with a second ending bracket starting at measure 82.



► These  
are  
the  
176  
bars

89. 90. 91. 92. 93. 94. 95. 96.

First system of musical notation, measures 89-96. It consists of a treble clef staff and a bass clef staff. The treble staff contains a melodic line with various rhythmic values and accidentals. The bass staff contains a bass line with some complex rhythmic patterns, including sixteenth-note runs, and first and second endings are indicated with '1.' and '2.' markings.

97. 98. 99. 100. 101. 102. 103. 104.

Second system of musical notation, measures 97-104. Similar to the first system, it features a treble and bass staff. The bass staff includes first and second endings for measures 100 and 102.

105. 106. 107. 108. 109. 110. 111. 112.

Third system of musical notation, measures 105-112. The bass staff continues with first and second endings for measures 107 and 110.

113. 114. 115. 116. 117. 118. 119. 120.

Fourth system of musical notation, measures 113-120. This system shows a more straightforward bass line without first or second endings.

121. 122. 123. 124. 125. 126. 127. 128.

Fifth system of musical notation, measures 121-128. The bass staff includes first and second endings for measures 123 and 127.

► These  
are  
the  
176  
bars

A musical score consisting of eight systems of two staves each (treble and bass clef). The score is numbered 129 through 176. The notation includes various rhythmic values, accidentals, and dynamic markings such as *h* and *h<sup>r</sup>*. The piece concludes with a double bar line at the end of the 176th bar.

► And these are the number tables

ZAHLENTAFEL.

TABLE de CHIFFRES.

Erster Theil.

Premiere Partie.

	A	B	C	D	E	F	G	H
2	96	22	141	41	106	122	11	30
3	32	6	128	63	146	46	134	81
4	69	95	158	13	153	55	110	24
5	40	17	113	85	161	2	159	100
6	148	74	163	45	80	97	36	107
7	104	157	27	167	154	64	118	91
8	162	60	171	53	99	133	21	127
9	119	54	114	50	140	86	169	94
10	98	142	42	156	75	129	62	123
11	3	87	165	61	135	47	147	33
12	54	130	10	103	28	37	106	5

Zweiter Theil.

Seconde Partie.

	A	B	C	D	E	F	G	H
2	70	121	26	9	112	49	109	14
3	117	39	126	56	174	18	116	83
4	66	139	15	132	73	58	145	79
5	90	176	7	34	67	160	52	170
6	25	143	64	125	76	136	1	93
7	138	71	150	29	101	162	23	151
8	16	155	67	175	43	168	89	172
9	120	88	45	166	51	115	72	111
10	65	77	19	82	137	38	149	8
11	102	4	31	164	144	59	173	78
12	85	20	108	92	12	124	44	131



# How it works?



- ▶ The Musikalisches Würfelspiel is an artistic creation in which Mozart composed not only a piano piece but a waltz generator.
- ▶ Proceeding to the composition, you have to roll two dice 16 times, adding the two dice on each roll.
- ▶ Write down all the results obtained.



► Look at the number tables.



Erster Theil.

Premiere Partie.

	A	B	C	D	E	F	G	H
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- ▶ Look at the number tables.



	I	II	III	IV	V	VI	VII	VIII
2	96	22	141	41	105	122	11	30
3	32	6	128	63	146	46	134	81
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10	65	77	19	82	137	38	149	8
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12	35	20	108	92	12	124	44	131

- ▶ The columns indicate the roll and the rows the sum of the two dice.

- ▶ Finally, select the bars and you will get a magical melody.



	I	II	III	IV	V	VI	VII	VIII
2	96	22	141	41	105	122	11	30
3	32	6	128	63	146	46	134	81
4	69	95	158	13	153	55	110	24
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11	102	4	31	164	144	59	173	78
12	35	20	108	92	12	124	44	131



- ▶ Finally, select the bars and you will get a magical melody.
- ▶ This is the song for the selected bars:



	I	II	III	IV	V	VI	VII	VIII
2	96	22	141	41	105	122	11	30
3	32	6	128	63	146	46	134	81
4	69	95	158	13	153	55	110	24
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# CHALLENGE #1

- ▶ Using the 'Mozart Dice simulator' and two dice, create your own melody. Share it with the others groups...

Mozart Dice Version 1.00  
Copyright 1998 by VisionSoft  
Written by S. Goodwin  
MIDI files played and programmed by John Chuang

Mozart Dice - (C) 1998 by VisionSoft

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	96	22	141	41	105	122	11	30	70	121	26	9	112	49	109	14
3	32	6	128	63	146	46	134	81	117	39	126	55	174	18	116	83
4	69	95	158	13	153	55	110	24	66	139	15	112	73	58	145	79
5	40	17	113	85	161	2	159	100	90	176	7	4	67	160	52	170
6	148	74	163	45	80	97	36	107	25	143	64	25	76	136	1	93
7	104	157	27	167	154	68	118	91	138	71	150	29	101	162	23	151
8	152	60	171	53	99	133	21	127	16	155	57	75	43	168	89	172
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Mozart Dice - Minuet Rules      Mozart Dice - Trio Rules

Create      Play      Stop      Export MIDI      Close

You can export the song

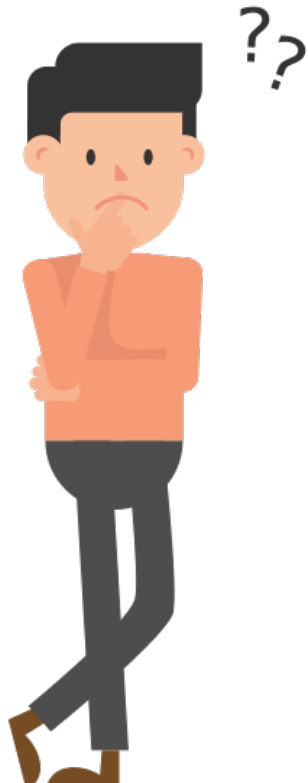
# Simple probability

- ▶ The probability of an event, like rolling an even number, is the number of outcomes that constitute the event (we call that “favourable outcomes”) divided by the total number of possible outcomes.



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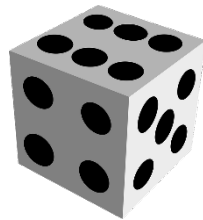
$$\textit{Probability} = \frac{\textit{favourable outcomes}}{\textit{possible outcomes}}$$

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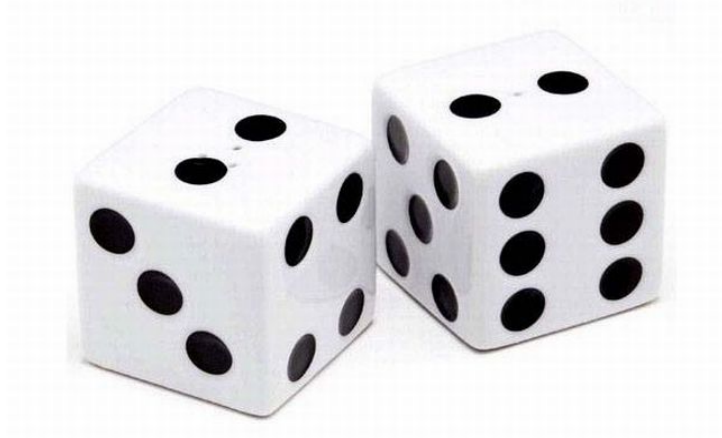
$$P(\textit{even number}) = \frac{3}{6} = \frac{1}{2}$$



# CHALLENGE #2

- ▶ Using the simple probability, calculate the probability when two dice are rolled. In groups...













I. Find out all the possibilities we can have when throwing two dice.



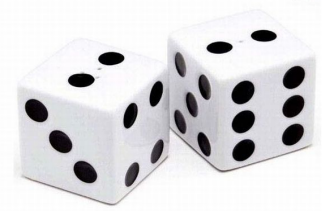
# CHALLENGE #2



## SAMPLE SPACE

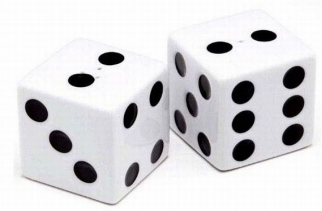
						
	(1 1)	(1 2)	(1 3)	(1 4)	(1 5)	(1 6)
	(2 1)	(2 2)	(2 3)	(2 4)	(2 5)	(2 6)
	(3 1)	(3 2)	(3 3)	(3 4)	(3 5)	(3 6)
	(4 1)	(4 2)	(4 3)	(4 4)	(4 5)	(4 6)
	(5 1)	(5 2)	(5 3)	(5 4)	(5 5)	(5 6)
	(6 1)	(6 2)	(6 3)	(6 4)	(6 5)	(6 6)

# CHALLENGE #2















II. Calculate the sum in each event

# CHALLENGE #2



II. Calculate the sum in each event












						
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12



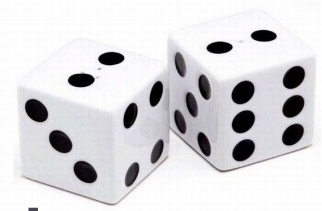
# CHALLENGE #2



III. Calculate the probability of each case

						
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

# CHALLENGE #2



III. Calculate the probability of each case

	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

$$P(2) = \frac{1}{36}$$

$$P(3) = \frac{2}{36} = \frac{1}{18}$$

$$P(4) = \frac{3}{36} = \frac{1}{12}$$

$$P(5) = \frac{4}{36} = \frac{1}{9}$$

$$P(6) = \frac{5}{36}$$

$$P(7) = \frac{6}{36} = \frac{1}{6}$$

$$P(12) = \frac{1}{36}$$

$$P(11) = \frac{2}{36} = \frac{1}{18}$$

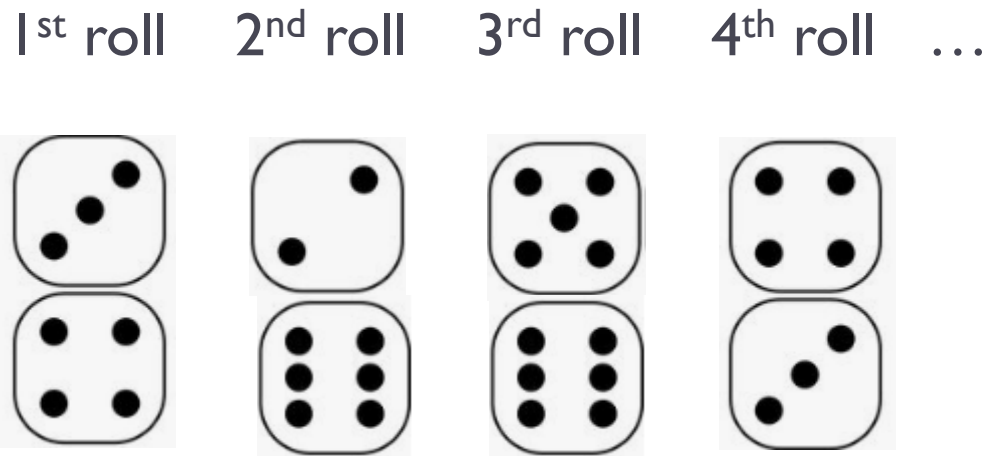
$$P(10) = \frac{3}{36} = \frac{1}{12}$$

$$P(9) = \frac{4}{36} = \frac{1}{9}$$

$$P(8) = \frac{5}{36}$$

# Independent events

- ▶ Two events, A and B, are independent if the fact that A occurs does not affect the probability of B occurring.
- ▶ In Mozart's game, each dice roll is an independent event:



# Probability of independent events

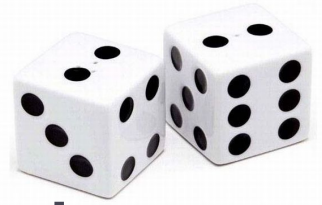
- ▶ To find the probability of two independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities.



$$P(A \text{ and } B) = P(A) \cdot P(B)$$

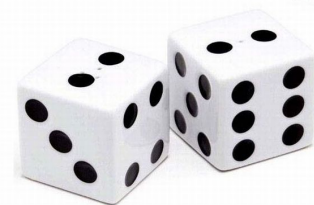


# CHALLENGE #3



As you can imagine, looking at the probabilities obtained before, there are some compositions that are more likely than others. But, can you find the melody that has the highest probability?

# CHALLENGE #3



Can you find the melody that has the highest probability?

	I	II	III	IV	V	VI	VII	VIII
2	96	22	141	41	105	122	11	30
3	32	6	128	63	146	46	134	81
4	69	95	158	13	153	55	110	24
5	40	17	113	85	161	2	159	100
6	152	60	171	33	99	133	21	127
7	104	157	27	167	154	68	118	91
8	132	60	171	33	99	133	21	127
9	119	84	114	50	140	86	169	94
10	98	142	42	156	75	129	62	123
11	3	87	165	61	135	47	147	33
12	54	130	10	103	28	37	106	5

	IX	X	XI	XII	XIII	XIV	XV	XVI
2	70	121	26	9	112	49	109	14
3	117	39	126	56	174	18	116	83
4	66	139	15	132	73	58	145	79
5	90	176	7	34	67	160	52	170
6	152	60	171	33	99	133	21	127
7	138	71	150	29	101	162	23	151
8	132	60	171	33	99	133	21	127
9	120	88	48	166	51	115	72	111
10	65	77	19	82	137	38	149	8
11	102	4	31	164	144	59	173	78
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# CHALLENGE #4



What is the probability of getting this melody?

	7	104	157	27	167	154	68	118	91		7	138	71	150	29	101	162	23	151
--	---	-----	-----	----	-----	-----	----	-----	----	--	---	-----	----	-----	----	-----	-----	----	-----

# CHALLENGE #4



What is the probability of getting this melody?

	7	104	157	27	167	154	68	118	91		7	138	71	150	29	101	162	23	151
--	---	-----	-----	----	-----	-----	----	-----	----	--	---	-----	----	-----	----	-----	-----	----	-----

$$P = \underbrace{\frac{1}{6} \cdot \frac{1}{6} \cdot \dots \cdot \frac{1}{6}}_{16 \text{ times}} = \left(\frac{1}{6}\right)^{16} = 3,54 \cdot 10^{-13}$$

16 times

# CHALLENGE #5



Find the probability of your composition?

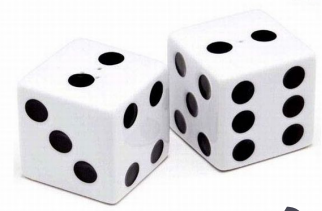
For example, look the probability of mine:

	I	II	III	IV	V	VI	VII	VIII
2	96	22	141	41	109	122	11	30
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# CHALLENGE #5



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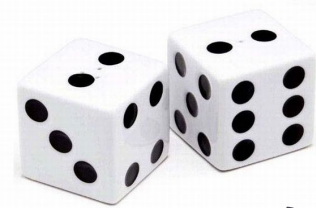
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$$\begin{aligned}
 P &= \frac{1}{6} \cdot \frac{1}{12} \cdot \frac{1}{6} \cdot \frac{1}{9} \cdot \frac{1}{36} \cdot \frac{5}{36} \cdot \frac{1}{12} \cdot \frac{1}{12} \cdot \\
 &= \frac{1}{9} \cdot \frac{5}{36} \cdot \frac{1}{6} \cdot \frac{1}{36} \cdot \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{5}{36} \cdot \frac{1}{9} = \\
 &= 5,276 \cdot 10^{-17}
 \end{aligned}$$

# CHALLENGE #5



Find the probability of your composition?

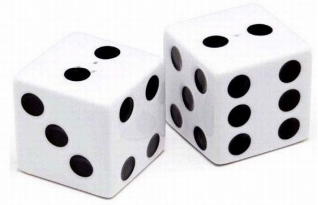
For example, look the probability of mine:

	I	II	III	IV	V	VI	VII	VIII
2	96	22	141	41	109	122	11	30
3	32	6	128	63	146	46	134	81
4	69	95	158	13	153	55	110	24
5	40	17	113	85	161	2	159	100
6	148	74	163	45	80	97	36	107
7	04	157	27	167	154	68	118	91
8	152	60	171	53	99	133	21	127
9	119	84	114	50	140	86	169	94
10	98	142	42	156	75	129	62	123
11	3	87	165	61	135	47	147	33
12	54	130	10	103	28	37	106	5

	IX	X	XI	XII	XIII	XIV	XV	XVI
2	70	121	26	9	112	49	109	14
3	117	39	126	56	174	18	116	83
4	66	139	15	132	73	58	145	79
5	90	176	7	34	67	160	52	170
6	25	43	64	125	76	136	1	93
7	138	71	150	29	101	162	23	151
8	16	155	57	175	43	168	89	172
9	120	88	48	166	51	115	72	111
10	65	77	19	82	37	38	149	8
11	102	4	31	164	144	59	173	78
12	35	20	108	92	12	124	44	131

$$\begin{aligned}
 P &= \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{36}\right)^3 \cdot \left(\frac{1}{9}\right)^3 \cdot \\
 &= \left(\frac{1}{12}\right)^5 \cdot \left(\frac{1}{18}\right)^0 \cdot \left(\frac{1}{36}\right)^2 = \\
 &= 5,276 \cdot 10^{-17}
 \end{aligned}$$

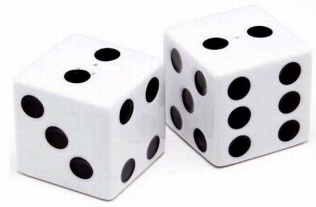
# CHALLENGE #6



Can you find out how many different melodies there are?



# CHALLENGE #6



Can you find out how many different melodies there are?



11<sup>16</sup>

# CHALLENGE #6



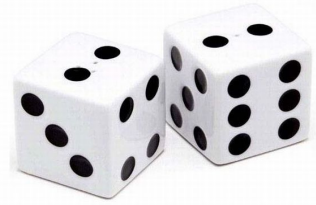
Can you find out how many different melodies there are?



45 949 729 863 572 161  
*Almost 46 quadrillion!!*

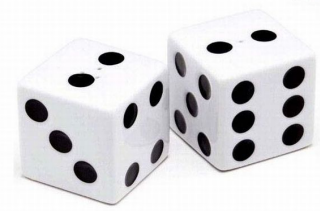


# CHALLENGE #7



Imagine that all the songs are played one after the other constantly, in a systematic order. Since each performance takes 30 seconds, how many years would it take us to listen to it, performing the play day and night without stopping?

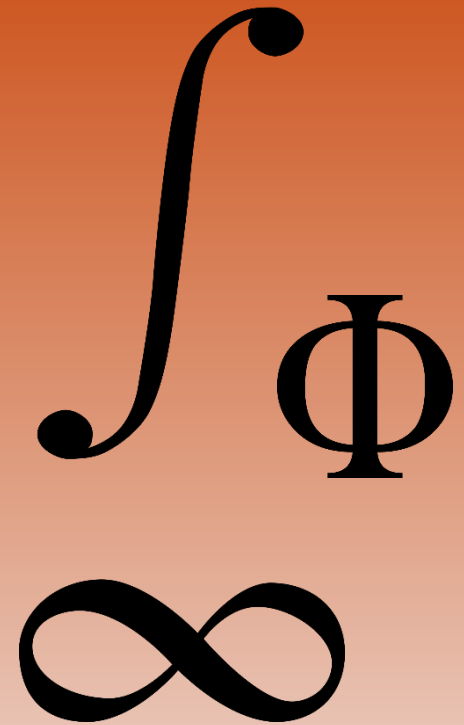
# CHALLENGE #7



Imagine that all the songs are played one after the other constantly, in a systematic order. Since each performance takes 30 seconds, how many years would it take us to listen to it, performing the play day and night without stopping?



43 711 691 270 .5 *years*  
*Almost 44 billions years!!*



**DANKE!**

**THANKS YOU SO MUCH FOR YOUR  
INTEREST AND ATTENTION**