UNIT 2. DIVISIBILITY

Eratosthenes was born in Cyrene, which is now in Libya in North Africa in 276 B.C. and he died in 194 B.C. in Alexandria (Egypt). He was not only a famous mathematician but also a well known geographer, astronomer, historian and poet. He studied in Athens and he was the librarian for the great Alexandrian University.

One of his accomplishments in mathematics was The Sieve of Eratosthenes, an ancient method for finding all primes numbers up to a specified number.
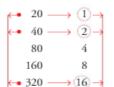
Eratosthenes is also known for his achievement in astronomy. Several astronomers and mathematicians before and after Eratosthenes tried to accurately measure the circumference of the Earth, but it was Eratosthenes that came through. He found the circumference of the Earth to be nearly 250,000 stadia (25,000 miles). It is also believed that Eratosthenes made a star catalog with approximately 675 stars and created a calendar that included leap years.

Egyptian-style divisions

Before the decimal numeral system existed, dividing wasn't so easy! Look, for example, at how Egyptians calculated 380 divided by 20.

They started by writing two columns:

— The first column had consecutive duplications of the divisor 20, without exceeding 380.

20 →	① →
40 →	② →
80	4
160	8
320 →	⑯ →
380	19 ←

— The second column had consecutive duplications of 1.

— From the first column, they took the necessary numbers to get to 380 → 20 + 40 + 320 = 380.

— Then, they took the corresponding numbers from the second column → 1 + 2 + 16 = 19.

— That result from the second column is the result of the division (the quotient): 380 : 20 = 19

1. Using the Egyptian system, divide 414 by 18.

PRIME NUMBERS RAP

LISTEN AND SING THAT INCREDIBLE SONG IN CLASS

With this song, students can learn prime numbers less tan 100

FOCUS on English

Don't get confused! In English we say 'prime number'. There is no such thing as a 'cousin number'.

https://youtu.be/cRz4hW9SPPc

- A **number** is **prime** if it has only two divisors: itself and the unit.
- A **number** is **composite** if it has more than two divisors.

VOCABULARY & EXPRESSIONS

Relation → relación

Divisibility → divisibilidad

Divisible by → divisible por

Divisible between → divisible entre

Multiple of → múltiplo de

Divisor of → divisor de

Exact division → división exacta

Remainder → resto

Prime number → número primo

Composite number → número compuesto

Divisibility criteria → criterio de divisibilidad

Factorising → fatorizar

Decompose a number into prime factors → descomponer un número en factores primos

Greatest common divisor (GCD) → máximo común divisor

Lowest common multiple (LCM) → mínimo común múltiplo

DIVISIBILITY

8 : 2 is an **exact division**.
↕
8 is **divisible** by 2.

8 is a **multiple** of 2. 2 is a **divisor** of 8.

Example

$$40 \quad | \underline{8}$$
$$0 \quad 5$$
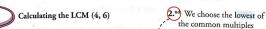
exact division

$$\rightarrow \quad 40 = 8 \cdot 5 \rightarrow$$

40 is divisible by 8

40 is multiple of 8

8 is divisor of 8

ACTIVITY: Do the division and write the relation of divisibility in the three posible ways

a) 36 y 9 b) 225 y 15 c) 575 y 23 d) 126 y 12 e) 108 y 3

Lowest common multiple

The smallest of the common multiples of two or more numbers (a, b, c...) is called the lowest common multiple. It is written as LCM (a, b, c...).

Optimal method Calculating the LCM (4, 6)

2.ⁿᵈ We choose the lowest of the common multiples

| 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

1.ⁿ We calculate the multiples of 4 and 6

3.ʳᵈ LCM (4,6) = 12

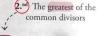| 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

ACTIVITY: Calculate the lowest common multiple:
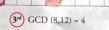
a) 8 and 10   b) 15 and 25
c) 12 and 9   d) 6 and 8

Greatest common divisor

The largest of the common divisors of two or more numbers (a, b, c...) is called the greatest common divisor. It is written as GCD (a, b, c...).

Traditional method Calculating the GCD (8, 12)

2.ⁿᵈ The greatest of the common divisors

| 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

1.ⁿ We calculate the divisors of 8 and 12

3.ʳᵈ GCD (8,12) = 4

| 12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

ACTIVITY: Calculate the greatest common divisor:

a) 8 and 10   b) 15 and 20
c) 12 and 16   d) 15 and 20

TRY AND DEDUCE

101 is the main player

• What happens to a two digit number if we multiply it by 101?

29 ₓ₅ᵧ 101 = ?

Try other numbers and check that this is always the case.

• What do all four-digit numbers formed by alternately repeating two digits have in common?

| 5 4 5 4 | | 8 7 8 7 |
| 1 3 1 3 | | 4 3 4 3 |

SOLUTION:

They are divisible by 101