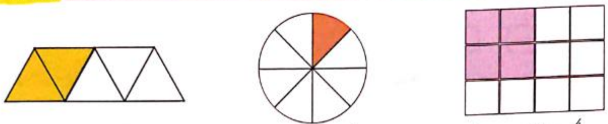
UNIT 4. Fractions

Fractions

Fractions

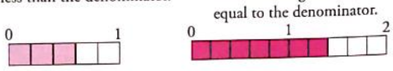


Two fifths $\frac{2}{5}$ One eighth $\frac{1}{8}$ Four twelfths $\frac{4}{12}$

Types of fractions:

Proper fraction: the numerator is less than the denominator.

Improper fraction: the numerator is greater than or equal to the denominator.



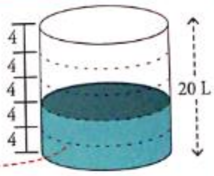
$\frac{3}{5}$ $\frac{7}{5}$

Fraction of a number

The barrel contains $\frac{2}{5}$ of 20 litres.

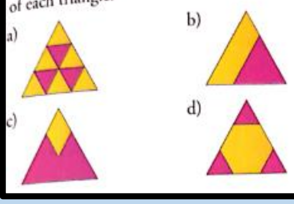
$\frac{2}{5}$ of 20 litres = $(20 : 5) \cdot 2 = 8$ litres

We divide the number by the denominator, then multiply by the numerator.



Write the fraction that represents the yellow sections of each triangle.

a) b) c) d)



Calculate.

a) $\frac{2}{5}$ of 15 b) $\frac{3}{4}$ of 12

d) $\frac{2}{3}$ of 30 e) $\frac{4}{5}$ of 30

g) $\frac{3}{4}$ of 48 h) $\frac{2}{3}$ of 72

Represent the following fractions.

a) $\frac{3}{5}$ b) $\frac{1}{3}$ c) $\frac{3}{4}$

Indicate if the following fractions are bigger, smaller or equal to the unit.

a) $\frac{2}{7}$ b) $\frac{3}{2}$ c) $\frac{6}{6}$ d) $\frac{8}{5}$ e) $\frac{3}{3}$ f) $\frac{5}{6}$

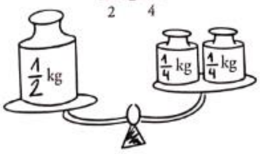
Equivalent fractions

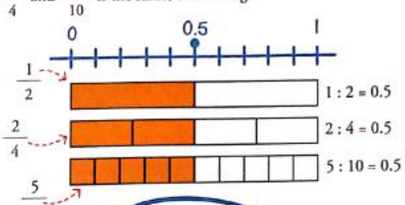
Different fractions with the same value:

The decimal value of the fractions $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{5}{10}$ is the same, even though the terms are different.

They are equivalent fractions.

$\frac{1}{2} = \frac{2}{4}$

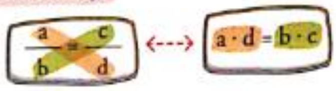




$\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$

How to determine whether two fractions are equivalent (related terms):

If two fractions are equivalent, the crossed products of their terms are the same.

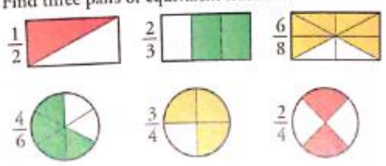


Check whether these are equivalent fractions.

a) $\frac{1}{2}$ and $\frac{3}{4}$ b) $\frac{2}{5}$ and $\frac{6}{15}$ c) $\frac{4}{6}$ and $\frac{6}{9}$

d) $\frac{6}{8}$ and $\frac{9}{11}$ e) $\frac{2}{12}$ and $\frac{3}{20}$ f) $\frac{20}{24}$ and $\frac{30}{36}$

Find three pairs of equivalent fractions.

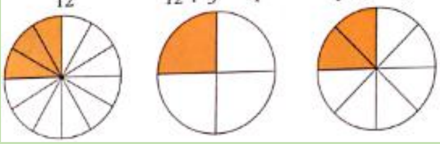


Obtain equivalent fractions / irreducible fraction

How to obtain equivalent fractions:

When we multiply or divide both terms of a fraction by the same number, the portion of a unit that the fraction represents does not change.

$\frac{3}{12}$ $\frac{3:3}{12:3} = \frac{1}{4}$ $\frac{1 \cdot 2}{4 \cdot 2} = \frac{2}{8}$ They are equivalent fractions.



$\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$

Write two equivalent fractions for each case.

a) $\frac{1}{4}$ b) $\frac{2}{3}$ c) $\frac{15}{20}$ d) $\frac{18}{24}$

Simplifying fractions:

To simplify a fraction, we replace it with another equivalent fraction that has simpler terms. We do this by dividing both terms by the same number.

$\frac{12}{18}$ ---> $\frac{12:2}{18:2} = \frac{6}{9}$ ---> $\frac{6:3}{9:3} = \frac{2}{3}$ ----> Irreducible fraction

Find the irreducible fraction.

a) $\frac{2}{4}$ b) $\frac{10}{14}$ c) $\frac{5}{15}$ d) $\frac{18}{22}$

e) $\frac{5}{25}$ f) $\frac{6}{27}$ g) $\frac{21}{28}$ h) $\frac{22}{33}$

i) $\frac{30}{45}$ j) $\frac{20}{60}$ k) $\frac{56}{80}$ l) $\frac{165}{330}$

Adding and subtracting fractions

Reducing to a common denominator

1.st step Calculate the LCM of the denominators.

2.nd step Divide the LCM into every denominator and multiply every numerator and every denominator by that number.

Result We get fractions that are equivalent to the initial fractions with the same denominator. We can now do calculations with them.

$\frac{a}{b}$ — a: numerator, b: denominator

Add:

$$\frac{1}{4}+\frac{3}{10}$$

1.st Reduce the fractions to a common denominator.

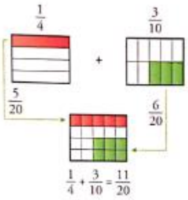
$$LCM\ (4,\ 10) = 20$$

2.nd Convert the fractions into equivalent fractions with the same denominator.

$$\frac{1\cdot5}{4\cdot5}+\frac{3\cdot2}{10\cdot2}$$

3.rd Calculate and solve.

$$\frac{1\cdot5}{4\cdot5}+\frac{3\cdot2}{10\cdot2}=\frac{5}{20}+\frac{6}{20}=\frac{5+6}{20}=\frac{11}{20}$$



Adding and subtracting fractions

1.st step Reduce to a common denominator.

2.nd step Calculate.

3.rd step Simplify the resulting fraction.

$$\frac{5}{6}+\frac{3}{7}=\frac{35+18}{42}=\frac{53}{42}$$

$$\frac{2}{3}-\frac{5}{6}=\frac{4-5}{6}=-\frac{1}{6}$$

a) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ b) $\frac{1}{4}+\frac{1}{5}+\frac{1}{10}$ c) $1-\frac{1}{2}-\frac{1}{5}$

a) $1+\frac{1}{5}$ b) $1-\frac{3}{5}$ c) $2+\frac{2}{7}$

Multiplying and dividing fractions

Multiplying fractions

To multiply fractions:

1.st step We multiply the numerators.

2.nd step We multiply the denominators.

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$$

a) $15\cdot\frac{3}{5}$ b) $12\cdot\frac{4}{3}$ c) $\frac{1}{2}\cdot\frac{1}{3}$

d) $\frac{5}{7}\cdot\frac{7}{5}$ e) $(-3)\cdot\frac{2}{9}$ f) $\frac{12}{5}\cdot\frac{5}{18}$

Calculate and, if possible, simplify.

a) $5:\frac{1}{2}$ b) $-\frac{1}{2}:5$ c) $\frac{3}{2}:6$

d) $(-3):\frac{6}{2}$ e) $\frac{2}{5}:\frac{4}{7}$ f) $\frac{12}{5}:\frac{6}{10}$

Dividing fractions

To divide fractions: cross multiply the terms.

$$\frac{a}{b}:\frac{c}{d}=\frac{a\cdot d}{b\cdot c}$$

Inverse fractions:

Two fractions are inverses of each other if their product is equal to 1.

$$\frac{2}{5}\cdot\frac{5}{2}=\frac{2\cdot5}{5\cdot2}=\frac{10}{10}=1$$

$\frac{2}{5}$ is the inverse of $\frac{5}{2}$

Combined operations

Combined operations

In combined operations with fractions, we have to follow the order of operations:

1.st step Calculations within brackets. [( )]

2.nd step Multiplication and division. × ÷

3.rd step Addition and subtraction. + −

$$1-\left(\frac{1}{2}-\frac{1}{3}\right)\cdot3$$
$$1-\frac{1}{6}\cdot3$$
$$1-\frac{1}{2}$$
$$\frac{1}{2}$$

Calculate.

a) $\frac{2}{3}\cdot2-\frac{5}{6}$ b) $\frac{2}{3}\cdot\left(2-\frac{5}{6}\right)$

c) $\frac{1}{6}:\frac{1}{2}-\frac{1}{6}$ d) $\frac{1}{6}:\left(\frac{1}{2}-\frac{1}{6}\right)$

e) $\frac{2}{3}+\frac{1}{6}\cdot\frac{3}{5}$ f) $\left(\frac{5}{3}-\frac{1}{6}\right):\frac{1}{2}$

VOCABULARY & EXPRESSIONS

Fraction → Fracción

Numerator → numerador

Denominator → denominador

a over b → a/b

Number line →recta numérica

Represent → representar

Graph → gráfica

Proper fraction → fracción propia

Improper fraction → fracción impropia

Equivalent fractions → fracciones equivalentes

Obtain→ obtener

Amplification → amplificación

Amplify → amplify

Simplification → simplificación

Simplify → simplificar

Irreducible fraction → gráfica

Reduce to a common denominator → reducir a común denominador

Lowest common multiple (LCD) → mínimo común múlitplo

Plus → más (signo +)

Minus → menos (signo -)

Compare → comparar

Order → ordenar

Biggest, greatest, largest → más grande

Smallest, lowest → más pequeño

Addition → suma

Add → sumar

Subtraction → resta

Subtract → restar

Bracket → paréntesis

Equal to → igual a

Compute → calcular, resolver

Solve → resolver

Perform → resolver

Multiplication → multiplicación

Multiplicate → multiplicar

Division → división

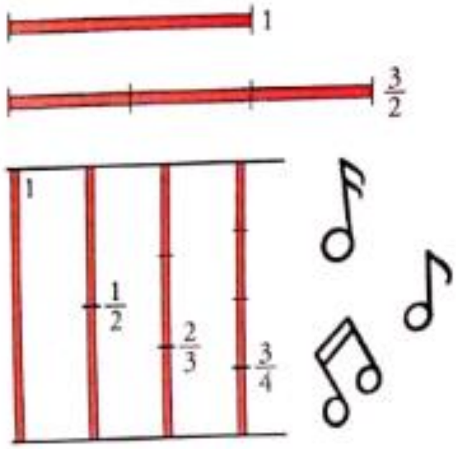
Divide → dividir

a times to b → a por b

a divided by b → a entre b

Combined operations → operaciones combinadas

READ AND LEARN



Fractions in ancient Greece

In ancient Greece (5th to 3rd century BCE), people did not think of fractions as numbers. Instead, they thought of them as relationships between natural numbers.

Since Pythagoras and his followers were interested in geometry, they used fractions to express the length of one segment in relation to another. They also discovered interesting connections between music and numbers:

If you pluck a string on a guitar, it makes a sound.

If you hold the string down in the middle and pluck it, it will make a sound that is in harmony with the one before. This also happens when you hold the string at points located at $\frac{2}{3}$ and $\frac{3}{4}$ of the total length.

This is how musical notes were invented.