UNIT 6. Algebra

Algebraic expressions

Match each statement to its algebraic expression.

a) The sum of a number and 3.
b) The difference between a number and 5.
c) Half of the sum of a number and 9.
d) A third of the difference between a number and 7.

i) $\frac{x+9}{2}$ ii) $x-5$ iii) $\frac{x-7}{3}$ iv) $x+3$

Algebraic expressions

In an algebraic expression the letters represent unknown or indeterminate information.

- Some examples of algebraic expressions:

$3x-5$ $\quad x^2+1$ $\quad \frac{(a+1)\cdot b}{5}$ $\quad \frac{(t+1)^2}{3}$ $\quad \frac{a+b}{a}$

The numerical value is the result of the expression when the letters are replaced by known numbers.

For $a=9$ and $b=3$ ⟶ $\frac{(a+1)\cdot b}{5} = \frac{(9+1)\cdot 3}{5} = 6$

Find the numerical value of the following algebraic expressions for these values of x:

$x=0 \quad x=1 \quad x=2 \quad x=-1 \quad x=2$

$a) 3x-2$ $\quad b) 5x+4$ $\quad c) 7-2x$
$d) -6x+10$ $\quad e) x^2-3x$ $\quad f) 2x-x$

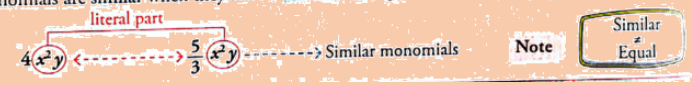
Find the numerical value of these algebraic expressions when the variables x and y have the values: $x=2 \quad y=-3$

$a) 3x-2y+5$ $\quad b) -x+4y-2$
$c) (x-2)\cdot(y+8)$ $\quad d) 3-2x+5\cdot(y-3)$

Monomials

A monomial is the product of a known number (coefficient) and one or more letters (literal part).

The degree of a monomial is the sum of the degrees of its letters.
Two monomials are similar when they have the same literal part (the same letters with the same exp



Indicate the coefficient, the literal part and the degree of these monomials

$a) 2a$ $\quad b) x^2$ $\quad c) -3ab$
$d) \frac{1}{2}xy^3$ $\quad e) -4x$ $\quad f) 6x^3y$
$g) xy$ $\quad h) -abc$ $\quad i) -3ab$
$j) -\frac{3}{5}ax^2z$ $\quad k) x^2yz$ $\quad l) 2-x$

Operations with monomials

Adding and subtracting monomials

Only similar monomials can be added or subtracted.

$5x-3x=2x$ Can be calculated $\quad x^2+x$ Cannot be calculat

Multiplying monomials

The product of two or more monomials is another monomial:

- $(3a)\cdot(2b)=3\cdot a\cdot 2\cdot b=3\cdot 2\cdot a\cdot b=6ab$
- $(-3x)\cdot 4x=(-3)\cdot x\cdot 4\cdot x=(-3)\cdot 4\cdot x\cdot x=-12x^2$

Dividing monomials

To divide, we use what we know about calculating with numbers, and we can get:

$3x:6x=\frac{3\cdot x}{6\cdot x}=\frac{3\cdot x}{2\cdot 3\cdot x}=\frac{1}{2}$ $\quad (-15ab^2):3ab=\frac{3\cdot(-5)\cdot a\cdot b\cdot b}{3\cdot a\cdot b}=-5b$ $\quad (-15a^2b):(-6ab^2)=\frac{(-3)\cdot 5\cdot a\cdot a\cdot b}{(-3)\cdot 2\cdot a\cdot b\cdot b}=\frac{5a}{2b}$

a number — a monomial — an algebraic fraction

The quotient is...

Perform the additions and subtractions of monomials:

$a) 2x+3x$ $\quad b) -4ab+2ab$
$c) -2x^2-4x^2$ $\quad d) -5xy^2+xy^2$
$e) 5x-7y+9x-y+2y+x$
$f) 3x^2+6x-7xy+3xy-2x^2$
$g) -y^2-9x+x-7y^2+5y^2+10$
$h) ab-4ab$ $\quad i) 2x-10x$
$j) -3x^2-8x^2$ $\quad k) 2x-2xy$

Multply or divide these monomials

$a) 2m\cdot(-3m)$ $\quad b) 4x^2\cdot 2x$
$c) 3a\cdot 4b^2$ $\quad d) 5x\cdot(-xy)$
$e) -2a^2\cdot(-5a)$ $\quad f) 3xy\cdot(-2x^2)$
$g) 6x:2x$ $\quad h) (-8xy):(-2x)$
$i) (-2a^2bc^3):(abc^2)$
$j) 15x^4:(-5x^2)$ $\quad k) 48x^2y^3:6xy^2$

Equations

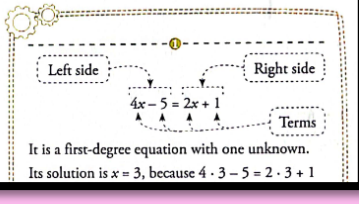
$3x - 4 = 8$ ----> The equality is only true for $x = 4$ ----------> **It is an equation**

$6x - 4x = 2x$ ----> The equality is true for any value of x ----------> **It is an identity**

Elements of an equation

To work with equations we first have to know which are their terms:

- Sides: The expressions that appear on either side of the equal sign.
- Terms: The addends on either side of the equation.
- Unknowns: The letters that appear in the terms.
- Solutions: The values of the letters that make the equality true.



It is a first-degree equation with one unknown.

Its solution is $x = 3$, because $4 \cdot 3 - 5 = 2 \cdot 3 + 1$

Deterrmine which of these equations have $x = 2$ as a solution

$a)\ 4 - x = 6$ $\quad b)\ 5 - 2x = 9$

$c)\ -x + 3 = 1 + x$ $\quad d)\ 6x - 4 = 4 + 5x$

$e)\ 3x + 7 = 1$ $\quad f)\ 8 = x - 3$

Draw a line to connect:

$3x^2 + 2x = 1$	First degree equation with one unkown
$-2x + 8 = 3x - 4x + 2$	Second degree equation with two unkowns
$3x + y = 8 + 3x$	Second degree equation with one unkown

Speaking. Discuss as a group which of these equalities are equations and which are identities.

a) $2 = x + 6$

b) $5x - 3x = 2x$

c) $7(9x - 1) + 6 = -3x - 1$

d) $2(2x + 4) - 6 = 4x + 2$

e) $3(4x - 2) + 7 = 7x - 19$

f) $3x + 2x = (3 + 2)x$

g) $5x - 3 = 4 - 7x$

Solving equations

EXAMPLE

6. Solve this linear equation.

$2x - 4 = 10$

The 4, which is subtracting in the first member, *goes* to the other member and is added to it.

$2x - 4 = 10 \rightarrow 2x = 10 + 4 \rightarrow 2x = 14$

The 2, which is multiplying in the first member, *goes* to the other member and divides it.

$2x = 14 \rightarrow x = \frac{14}{2} = 7$

Solve the equations and verify the solution.

a) $4x + 1 = 5x - 8$

b) $5 + 6x = -4 - 3x$

c) $10y - 3y + 3 = -18$

d) $-x - 5 = 3x + 19$

e) $-7 + 3y = y - 3$

f) $-8x + 6 = -1 - x$

g) $12 = 5x - 9 - 2x$

h) $30 + 8y = -7y$

Solve these equations.

a) $x + 7 = 9$

b) $4x - 12 = 3x$

c) $x - 5 = 2x - 4$

d) $3x - 5 = -5 - 7x$

a) $x + 2 = 5$

b) $4 + y = 8$

c) $11 = z + 7$

d) $-2 + a = 2$

e) $c - 7 = 12$

f) $20 = -y + 4$

g) $-3 + b = 7$

h) $6 - c = 9$

Solve these linear equations.

a) $x - 6 = 2$

b) $6x = -6$

c) $3 - x = 1$

d) $3x + 2 = 11$

e) $5x - 2 = 3$

f) $10x - 2 = -22$

Solve these equations and check the solution

$a)\ -3x - 2 = 7$

$b)\ 3x + 12 = -9 - 4x$

$c)\ -2 + x = 4x - 2 - 6 - x$

$d)\ 1 - x = 6$

$e)\ 3x + 6 = 3$

$f)\ -x - 5 + 2x = -10 + 5x + 9$

$g)\ -2 - 2x = -4$

$h)\ -3x = -5x + 8 + 6$

a) $2x + 7 = 3$

b) $16 = -5 + 7b$

c) $6 = -2x$

d) $-3a - 8 = 4$

e) $-4 + 3a = 5$

f) $5 - 4y = 1$

g) $-5 = -c - 3$

h) $-6z - 9 = 15$

More equations

KNOW HOW TO

Solve equations with brackets

Solve this linear equation that contains brackets.

$4 \cdot (x - 6) - 2 = 5 - 3 \cdot (x + 1)$

Follow these steps

1. Expand the brackets.

$4 \cdot (x - 6) - 2 = 5 - 3 \cdot (x + 1)$

$4x - 24 - 2 = 5 - 3x - 3$

2. Group the terms with the unknown in one member and the numerical terms in the other.

Group all terms with unknowns in the first member.

$4x + 3x = 5 - 3 + 24 + 2$

3. Simplify the similar terms, if there are any.

$7x = 28$

4. Make x the subject and find its numerical value.

The 7, which is multiplying in the first member, *goes* to the second member and divides it.

$x = \frac{28}{7} = 4$

The solution of the equation is $x = 4$.

If the bracket is preceded by the sign −, when expanding it, we invert all the signs of the addends inside the bracket.

$3 - 2(x + 1) = 3 - 2x - 2$

$3 - 2(x - 1) = 3 - 2x + 2$

a) $6 \cdot (a - 5) - 2 = 2a$

b) $-4 \cdot (3 - x) + 9 = x$

c) $(4 + 3x) \cdot (-7) + 2 = -8x$

d) $5a = 2 \cdot (6 - 3a) + 21$

e) $3 - (b + 4) = 0$

f) $9c - 5 \cdot (c - 1) = 1$

g) $3 - 2 \cdot (y - 4) = 11$

h) $16 - 4y = 5 - 7(2 - 3y)$

a) $2x + 4 \cdot (5 - 2x) = 8$

b) $7x - 3 \cdot (x + 1) = -1$

c) $5x - 3 \cdot (9 - x) = -3$

d) $6x + 5 \cdot (2 - 3x) = 4$

e) $x - 3 \cdot (x - 2) = 10$

f) $2x + 7 \cdot (x + 4) = 19$

VOCABULARY & EXPRESSIONS

Algebraic expression → expresión algebraica

Letter → letra

Mathematical operations → operaciones matemáticas

Numerical value → valor numérico

Substitute the letters → sustituir las letras

Plus, minus → más, menos

Sum → suma

Difference → diferencia

Twice, triple → doble, triple

Half, third part → mitad, tercera parte

Square of → cuadrado de

Monomial → monomio

Coefficiente →coeficiente

Literal part → parte literal

Variable → variable

Degree of the monomial → grado del monomio

Similar monomials → monomios semejantes

Addition → suma

Add → sumar

Subtraction → resta

Subtract → restar

Equal to → igual a

Solve, perform, compute, calculate → resolver/calcular

Multiplication → multiplicación

Multiply → multiplicar

Division → división

Divide → dividir

Multiplied by → multiplicado por

Divided by → dividido entre

Equation → ecuación

Members → miembros

Terms → términos

Unknowns → incógnitas

Solution → solución

Check the solution → comprobar la solución

Make the equality true → hacer la igualdad cierta

Solve an equation → resolver una ecuación

Equation of degree one → ecuación de grado 1

Linear equation → ecuación lineal

Equation with brackets → ecuación con paréntesis

INVESTIGATE

Below we explain a game for two people.

Play it, study the results and explain the winning strategy!!!

The game begins by placing a token on the START position. When it is a player's turn, they move the token to one of the adjacent spots below. The player who gets the token to the FINISH position first wins.

HELP:

- Play several times.
- Try it with boards that have fewer points.
- From what positions are you guaranteed to win?
- Do you prefer to start first or second?