Why logarithms?

Several centuries before the emergence of calculators, logarithms were invented to cope with the enormous operations that had to be done by hand. How? This was done by converting products and quotients into additions and subtractions (which are operations that are much easier to work with than others). To do this, they had to use enormous tables (in very thick books), which contained the logarithms of the factors.

Now that we have calculators, why do we need logarithms? Well, it's mostly a question of culture, but also because we will encounter them in algebraic simplifications and functional expressions in the world of science and technology.

USE OF THE LOGARITHMS

The **logarithm base 10** is called the common logarithm and is commonly used in science and engineering. Some examples of this include sound (decibel measures), earthquakes (Richter scale), the brightness of stars, and chemistry (pH balance, a measure of acidity and alkalinity).

The **natural logarithm** has the number e as its base; its use is widespread in mathematics and physics, because of its simpler integral and derivative.

The **binary logarithm** uses base 2 and is commonly used in computer science.

RICHTER SCALE

A logarithmic function that is used to measure the magnitude of earthquakes. The magnitude of an earthquake is related to how much energy is released by the quake. Instruments called seismographs detect movement in the earth; the smallest movement that can be detected shows on a seismograph as a wave with amplitude A_0.

The Richter scale measure of the magnitude of the earthquake using the formula:

$$R = \log\left(\frac{A}{A_0}\right)$$

A – the measure of the amplitude of the earthquake wave

A_0 – the amplitude of the smallest detectable wave (or standard wave)

Example

Problem **An earthquake is measured with a wave amplitude 392 times as great as A_0. What is the magnitude of this earthquake using the Richter scale, to the nearest tenth?**

$R = \log\left(\frac{A}{A_0}\right)$ Use the Richter scale equation.

$R = \log\left(\frac{392A_0}{A_0}\right)$ Since A is 392 times as large as A_0, $A = 392A_0$. Substitute this expression in for A.

$R = \log 392$
$R = 2.5932\ldots$
$R \cong 2.6$

Simplify the expression $\left(\frac{392A_0}{A_0}\right) = 392$.

Use a calculator to evaluate the logarithm.

Answer The magnitude of this earthquake is 2.6 on the Richter scale.

VOCABULARY & EXPRESSIONS

- **POWER:** potencia
- **Base:** base
- **Exponent:** exponente
- **RADICAL:** radical
- **Index:** índice
- **Radicand:** radicando
- **Root:** raíz
- **Reduction to a common index:** reducción a índice común
- **Equivalent radicals:** radicales equivalentes
- **Withdrawing factors from radical:** extraer factores de un radical
- **Rationalisation:** racionalización
- **Square root:** raíz cuadrada
- **Cube root:** raíz cúbica
- **Exact root:** raíz exacta
- **LOGARITHM:** logaritmo
- **Base:** base
- **Base change:** cambio de base
- **Decimal or common logarithm:** logaritmo decimal
- **Natural logarithm:** logaritmo neperiano

LOGARITHMS

In mathematics, the **logarithm** is the inverse function to exponentiation. That means the logarithm of a given number x is the exponent to which another fixed number, the base b, must be raised, to produce that number x. In the simplest case, the logarithm counts the number of occurrences of the same factor in repeated multiplication.
For example:

The equality $2^3 = 8$ can also be written: $log_2\ 8 = 3$.
$log_2\ 8$ is read 'base-2 logarithm of 8".

In words we say: $\log_a b \rightarrow Base - a\ logarithm\ of\ b$

Doing examples:

$\log_5 10 \rightarrow Base - 5\ logarithm\ of\ 10$

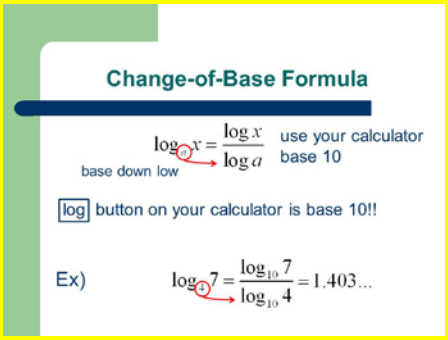
$Base - 3\ logarithm\ of\ 6 \rightarrow \log_3 6$

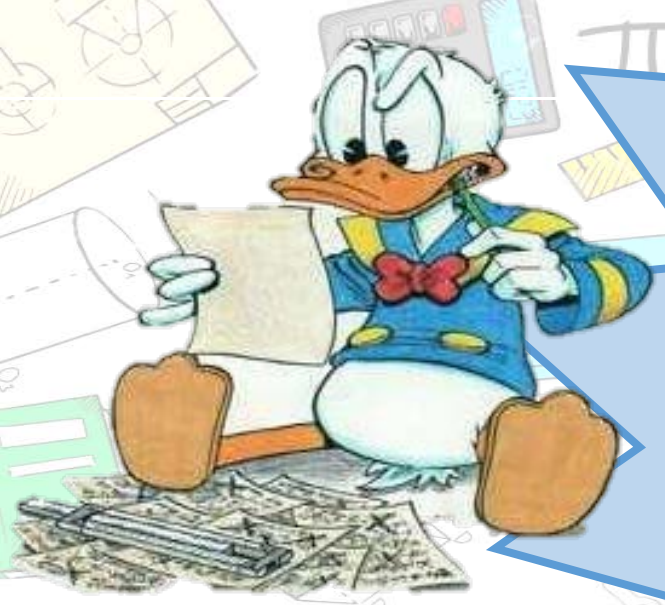
Invent more examples by yourself.

LOGARITHMS WITH CALCULATOR

a) If you have in your calculator the button $\log_{\square}\square$

b) If you haven´t it: you have to do the base change (base 10 or e) and use the button log or ln





WE ARE GOING TO PRACTICE

Use a calculator to find:

a) $log_2\ 1\,024$

b) $log_5\ 300$

Solution

a) $log_2\ 1\,024 = \frac{log_{10}\ 1\,024}{log_{10}\ 2}$; log 1 024 ÷ log 2 = 10 → $log_2\ 1\,024 = 10$

b) $log_5\ 300 = \frac{log_{10}\ 300}{log_{10}\ 5}$; log 300 ÷ log 5 = 3.54... → $log_5\ 300 = 3.54\ldots$