UNIT 4. EQUATIONS (4º ESO)

MAIN COMPONENTS OF AN EQUATION

a) $\underbrace{\overbrace{7x + z}^{\text{1st member}}}_{\text{terms}} = \overbrace{8x + 9}^{\text{2nd member}}$ ← independent term

Unknowns: x, z Degree: 1

b) $\underbrace{\overbrace{x^2 - 2x}^{\text{1st member}}}_{\text{terms}} = \overbrace{3}^{\text{2nd member}}$ ← independent term

Unknown: x Degree: 2

Identify the main components of these equations:

$\boldsymbol{a})\ 4x - 7z = 5x - 10 \qquad \boldsymbol{b})\ 4x^2 - 7x = 3x - 8$

TRANSFORMATIONS THAT MAINTAIN THE EQUIVALENCE IN EQUATIONS

TRANSFORMATION	PRACTICAL RULE
Adding or subtracting the same expressions in the two sides of the equality.	The summand in one side becomes the subtracted amount in the other, and vice versa.
Multiplying or dividing the two sides of the equality by the same number (not zero).	The number multiplying in the left side becomes the divisor of the right, and vice versa.

LINEAR EQUATIONS

To solve these equations we use the practical rules:

- $3x + 7 = 5x - 5 + 2x \rightarrow 3x - 5x - 2x = -7 - 5 \rightarrow -4x = -12 \rightarrow$
$$x = \frac{-12}{-4} = 3$$
- $2(x-4) + 3x = 5(2x+3) \rightarrow 2x - 8 + 3x = 10x + 15 \rightarrow$
$$2x + 3x - 10x = +8 + 15 \rightarrow -5x = 23 \rightarrow$$
$$x = \frac{23}{-5} = -\frac{23}{5}$$

Solve these equations:

$\boldsymbol{a})\ 3x + 5(3 - 4x) = 5x - 4 \qquad \boldsymbol{b})\ 10x + 3 = 5 - 7x + 3x$

$\boldsymbol{c})\ 3 - (x + 12) = 7(2x - 30) + 54 - 3x$

VOCABULARY & EXPRESSIONS

Equations: Ecuaciones

Identity: identidad

Members: miembros

Terms: términos

Unknowns/variables: variables

Coefficients: coeficientes

Degree: grado

Solutions: soluciones

Solving an equation: resolver una ecuación

Equivalent: equivalente

Transposing terms: trasponer términos

Linear equation: ecuación lineal

First degree equation: ecuación de primer grado

Quadratic equation: ecuación cuadrática

Second degree equation: ecuación de segundo grado

Complete: completa

Incomplete: incompleta

Discriminant: discriminante

Ruffini's rule: regla de Ruffini

SECOND DEGREE EQUATIONS OR QUADRATIC EQUATIONS

A **quadratic equation** with one unknown is an equality between two algebraic expressions that can be written as $\boldsymbol{ax^2+bx+c=0}$ where a, b and c are real numbers and a ≠ 0.

If b and c are numbers different to 0, the equation is **complete**. In other case, it is **incomplete**.

INCOMPLETE QUADRATIC EQUATIONS

- If b = 0, the equation is $\boldsymbol{ax^2+c=0}$
  There are two solutions or there are no solutions.
- If c = 0, the equation is $\boldsymbol{ax^2+bx=0}$.
  There are two solutions.
- If b = 0 and c = 0, the equation is $\boldsymbol{ax^2=0}$
  There are one solution and it is always zero.

COMPLETE QUADRATIC EQUATIONS

To solve these equations we use the formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

HOW MANY SOLUTIONS HAS AN EQUATION?

We use the **discriminant** which is:

$$\Delta=b^2-4ac$$

- If Δ > 0, the equation has two solutions.
- If Δ = 0, the equation has only one equation
- If Δ < 0, the equation has no solutions

Calculate the number of solutions of:

$$x^2-3x-10=0$$

$a=1 \qquad b=-3 \qquad c=-10$

$$\Delta=b^2-4ac=(-3)^2-4\cdot1\cdot(-10)=$$
$$=9-(-40)=49>0$$

The equation has two solutions:

Calculate the number of solutions of:

a) $x^2-4x+3=0$
b) $2x^2+4x+4=0$
c) $7x^2+10=0$

QUADRATIC EQUATIONS

Solve the quadratic equation $x^2+2x-8=0$.

Identify a, b and c: $\quad a=1 \quad b=2 \quad c=-8$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{2^2-4\cdot1\cdot(-8)}}{2\cdot1}=\frac{-2\pm\sqrt{4-(-32)}}{2}=$$

$$=\frac{-2\pm\sqrt{36}}{2}=\frac{-2\pm6}{2}\rightarrow\begin{cases}x_1=\dfrac{-2+6}{2}=2\\[2ex] x_2=\dfrac{-2-6}{2}=-4\end{cases}$$

The equation has two solutions: $x_1=2$ and $x_2=-4$.

Solve these equations:

$\boldsymbol{a})\ x^2-7x+12=0 \qquad \boldsymbol{b})\ -2x^2+6x-3=0$

$\boldsymbol{c})\ x^2-49=0 \qquad \boldsymbol{d})\ 2x^2-8x+8=0$