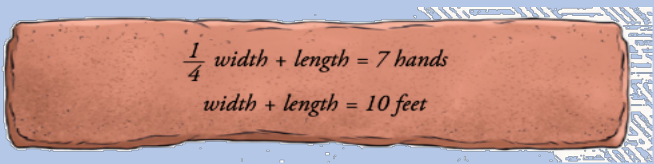
UNIT 7. SYSTEMS OF EQUATIONS (4º ESO)

HISTORY

Mathematicians inf Babylon studied both equations and system of equations. However, they were much more interested in the latter. They devised and solve linear systems of equations with several unknowns. They called these unknowns *length, width, volume, area…* even if they had nothing to do with geometry.

In Babylon, they solved problems using guesswork. They tried different amounts until they got the right solution. The following problem appears on one of their tablets:



Diophantus devised algebraic problems that could easily be made into systems of equations. He solved them by carefully choosing and naming an unknown that allowed him to create one single equation.

VOCABULARY & EXPRESSIONS

- ⇒ **Equation**: Ecuación
- ⇒ **System of equations**: sistema de ecuaciones
- ⇒ **Coefficient**: coeficiente
- ⇒ **Unknowns**: incógnitas
- ⇒ **Independent term**: término independiente
- ⇒ **Degree**: grado
- ⇒ **Solve**: resolver
- ⇒ **Solution**: solución
- ⇒ **Inconsistent system**: Sistema incompatible
- ⇒ **Independent system**: sistema compatible determinado
- ⇒ **Dependent system**: Sistema compatible indeterminado
- ⇒ **Elimination method**: método de reducción
- ⇒ **Equalisation method**: método de igualación
- ⇒ **Substitution method**: método de sustitución

SYSTEMS OF LINEAR EQUATIONS

Two equations form a system of equations when we work with them to find their common solution.

When two equations form a system, we write them like this:

$$\begin{cases} ax + by = c \\ a'x + b'y = c' \end{cases}$$

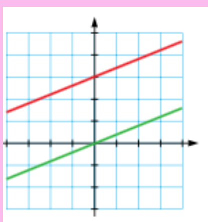
The **solution** to a system is the common solution to both equations.

NUMBER OF SOLUTIONS OF A SYSTEM OF LINEAR EQUATIONS

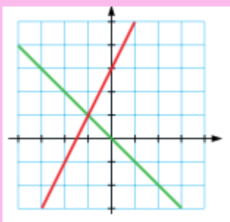
The number of solutions of a system of linear equations is:

- **Zero solutions.** The graphic representation of this is two parallel straight lines. This system is called **inconsistent system**.
- **One solution**. The graphic representation of this is two straight lines that intersect at a single point. This system is called **independent system**.
- **Infinite solutions.** The graphic representation of this is two coincident straight lines. This system is called **dependent system**.

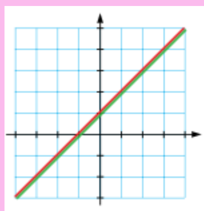
0 solutions



1 solutions



Infinite solutions



EQUALISATION METHOD

Solve the system using the equalisation method:

$$\begin{cases} 5x + 12y = 6 \\ 3x + 2y = 2 \end{cases}$$

1. Isolate x in each of the equations $x = \dfrac{6-12y}{5},\ x = \dfrac{2-2y}{3}$
2. Equalise the two equations: $\dfrac{6-12y}{5} = \dfrac{2-2y}{3}$
3. Solve the resulting equation
   $$3(6-12y) = 5(2-2y) \rightarrow 18 - 36y = 10 - 10y \rightarrow$$
   $$\rightarrow -36y + 10y = 10 - 18 \rightarrow -26y = -8 \rightarrow$$
   $$\rightarrow y = \frac{-8}{-26} = \frac{4}{13}$$
4. Substitute the y value into any of the expressions in step 1
   $$x = \frac{2 - 2\cdot\left(\frac{4}{13}\right)}{3} = \frac{6}{13}$$
5. We get the solution: $x = \dfrac{6}{13},\ y = \dfrac{4}{13}.$

SUBSTITUTION METHOD

Solve this system using the substitution method:

$$\begin{cases} 5x + y = -2 \\ 3x + 2y = 3 \end{cases}$$

1. Isolate y in the first equation: $y = -2 - 5x$
2. Substitute the expression for y in the second equation: $3x + 2(-2-5x) = 3$
3. Solve the resulting equation
   $$3x - 4 - 10x = 3 \rightarrow -7x = 7 \rightarrow x = -1$$
4. Substitute the x value into the equation of step 1
   $$y = -2 - 5x \rightarrow y = -2 - 5\cdot(-1) = -2 + 5 = 3$$
5. We get the solution: $x = -1, y = 3.$

REDUCTION METHOD

Solve these systems using the reduction method:

a) $\begin{cases} 5x + 7y = -1 \\ 3x - 7y = 33 \end{cases}$ b) $\begin{cases} 2x + 5y = 10 \\ 7x + 10y = 20 \end{cases}$

a) First system of equations

1. Add the equations together to make y disappear: $8x = 32 \rightarrow x = 4$
2. Substitute the value of x in the first equation: $5\cdot 4 + 7y = -1 \rightarrow y = -3$
3. We get the solution: $x = 4,\ y = -3$

b) Second system of equations

1. We multiply the first equation by -2 to get the same coefficients of y, but with different signs: $-4x - 10y = -20$
2. Add the two equations to make y dissapear:
   $$\begin{cases} -4x - 10y = -20 \\ 7x + 10y = 20 \end{cases}$$
   $$3x \quad = 0 \rightarrow x = 0$$
3. Substitute the x value into one equation: $2\cdot 0 + 5y = 10 \rightarrow y = 2$
4. We get the solution: $x = 0,\ y = 2$

Solve these problems using the different methods of resolution

a) $\begin{cases} 3x + 5y = 11 \\ 4x - 5y = 38 \end{cases}$ b) $\begin{cases} x + 3y = 5 \\ 5x + 7y = 13 \end{cases}$

c) $\begin{cases} x - 4y = 11 \\ 5x + 7y = 1 \end{cases}$ d) $\begin{cases} 6x + 3y = 0 \\ 3x - y = 3 \end{cases}$

CAN YOU SOLVE THIS RIDDLE?